Merely Statistical Evidence and the Principle of Indifference

Introduction

Here is a classic case of "merely statistical evidence" from civil law, first introduced to the legal epistemology literature by Thomson (1986) and based on *Smith v. Rapid Transit*.

> **Buses:** A bus crashes into a pedestrian. 100 buses operate on the route. 80 are operated by the Blue Bus company and 20 are operated by the Red Bus company. The crash was seen by one eyewitness, but she was not able to tell which company the bus was from.

Here's what people say about this case:[1] the evidence in *Buses* supports a high probability (.8) that the Blue Bus company is liable, but this evidence against the Blue Bus company is not enough to find them guilty. This is a puzzle. Civil law uses a "preponderance of the evidence" standard, and evidence that supports a .8 probability that a defendant is guilty (e.g., by eyewitness testimony) often meets this standard. But the evidence in this case does not meet the standard. Psychologists have noticed that ordinary people are reluctant to convict on this evidence (they call it the "Wells' effect"), it's the consensus view of many legal epistemologists, and it's also borne out in the law. It's so natural to think *it's wrong to hold the Blue Bus company responsible just because they have more buses!* So, a vast literature studies cases like *Buses*, trying to figure out what's missing in the cases of merely statistical evidence.

I think that the literature rests on a mistake, at least insofar as it's motivated by cases like *Buses*. It's much too quick to say that the evidence supports a .8 probability that the Blue Bus Company is liable. In fact, in some plausible ways of spelling out the details of the case, the evidence (counterintuitively) supports a high probability that the *smaller* bus company, the Red Bus Company, is responsible for the crash.

Here's how this paper will proceed. In §I, I'll argue that the idea that the evidence in *Buses* supports a high probability that the Blue Bus company comes from what may appear to be a

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[1] [Insert citations for some of the ~1000 papers that reference the problem of merely statistical evidence and motivate the problem using the *Buses* case.]

straightforward use of the Principle of Indifference but is in fact a misguided use. In §II, I'll argue that when we consider the plausible tacit assumptions in the background of the *Buses* case, the correct use of the Principle of Indifference will lead us to a probability that the Blue Bus company is guilty that are lower than .5. In §III, I'll argue that correctly applying the the Principle of Indifference can clear up some of the potential inconsistency in the use of statistical evidence in the law. Finally, I'll conclude with the idea that once we let go of the *Buses* case, more solutions to the problem of merely statistical evidence open up.

I. Buses and the Principle of Indifference

Here's the argument that the evidence in *Buses* supports a .8 probability that the Blue Bus company caused the crash, closely paraphrased from Thomson (1986):

P1. 100 buses operate on a route. 1 bus crashed.
P2. The probability that any of the buses crashed is .01.
P3. The Blue Bus company operated 80 of the buses.
C. The probability that a Blue Bus crashed is .8.

P1 and P2 are part of the set-up of the case. P2 is questionable. It must follow from indifference reasoning, along the lines of the following (White, 2010):

> **Principle of Indifference (POI):** If propositions p and q are evidentially symmetrical (the evidence no more supports one than the other), then P(p) = P(q).

P2 may appear to be a straightforward application of the POI. The evidence no more supports that one bus crashed than any other. 1 out of 100 buses crashed. So, the probability that any given bus crashed is .01. The Blue Bus company owns 80 of the buses, so the probability that a Blue Bus crashed is .8. Thus, there is a .8 probability that the Blue Bus company is responsible for the crash.

Here's the problem. Straightforward applications of the POI are few and far between. Here's another case, where similar reasoning seems absurd.

> **Books:** A library book has been destroyed. Pen scribbles are on each page. The detective narrows the suspects down to two library patrons: Annie and Bob. The detective searches their backpacks. Annie owns 80 pens and Bob owns 20 pens.

The similar reasoning goes like this. The evidence no more supports that one pen was used to scribble than any other. 1 out of 100 pens were used to scribble on the books. So, the probability that any given pen was used to scribble is .01. Annie owns 80 pens. So, the probability that one of Annie's pens was used to scribble on the book is .8. Thus, there is a .8 probability that Annie scribbled on the book.

In the *Books* case, it's a natural to think *it's wrong to hold Annie responsible just because she has more pens*! It's not a puzzle why this evidence isn't enough to hold Annie responsible. The evidence only supports a .5 probability that Annie did it.

The *Buses* case can come to resemble *Books*, by adding one bit of information:

> **Buses (Murder):** A bus crashes into a pedestrian. 100 buses operate on the route. 80 are operated by the Blue Bus Company and 20 are operated by the Red Bus Company. The crash was seen by one eyewitness, but she was not able to tell which company the bus was from. *The bus crash was a result of a murder plot by the company's CEO.*

Think about the reasoning from before: the evidence no more supports that one bus crashed than any other. 1 out of 100 buses crashed. So, the probability that any given bus crashed is .01. The Blue Bus company owns 80 of the buses, so the probability that a Blue Bus crashed is .8. Thus, there is a .8 probability that the Blue Bus company is responsible for the crash.

This seems absurd. The probability that the Blue Bus CEO murdered the pedestrian should be .5, not .8. It's natural to think: *it's wrong to hold the Blue Bus Company responsible just because they have more buses*. It's not a puzzle. In the *Buses Murder* case, the problem with the evidence against the Blue Bus company is not that it's merely statistical evidence, it's that it's not even evidence that they are responsible for the crash. If we have a case like *Murder* quietly in the back of our minds, then this provides a straightforward explanation for why the evidence does not suffice for a conviction. Of course, the idea that the bus crashed as part of a murder plot is far-fetched, but there are many more plausible ways in which the case could be elaborated upon to have this same result. Maybe the crash was a result of an act of gross negligence: a publicity stunt gone wrong, a driver who's woefully incompetent, or the use of a bus that's clanking and smoking with broken brakes, on the verge of a mechanical failure. In any of these cases, one should remain 50/50 that a Blue Bus crashed.

Where does the application of the POI go wrong, in the *Buses Murder* case? Huemer's (2009) "explanatory priority proviso" to the POI is instructive. The idea can be illustrated with the following case:

> *Marbles*. A lamp is either on or off. Annie draws a marble from a bag containing only red, blue, and green marbles. If she draws a blue marble, then she turns the lamp on. If she draws a blue or a green marble, then she turns the lamp off.

What is the probability that the lamp is on? On one apparently straightforward application of the POI, it is 1/2. The evidence no more supports that the lamp is on than it is off, so both have the same probability. On another apparently straightforward application of the POI, it is 1/3. The evidence no more supports that a red, blue, or green marble was drawn, so all three have the same probability. So, the probability that a red marble was drawn, and the lamp is on is 1/3.

The POI gives inconsistent answers, but the answer it should give is clear. The probability that the lamp is one is 1/3. One should assign equal probability to the colors of the marbles drawn, and from there figure out the probability that the lamp is on or off. Why? According to Huemer (2009), it's because the marble that is drawn from the bag is *explanatorily prior* to whether or not the lamp is on or off. The color of the marble causally determines whether the lamp is on or off. One should assign equal probabilities over the partitions of alternatives that are the most explanatorily fundamental, where this encompasses a range of explanatory relations, including *grounding, supervenience, temporal priority, and causal priority*.

In the *Books* case, whether Annie or Bob chose to scribble on the book is explanatorily prior to which pen they chose. Either Annie or Bob chose to scribble on the book, and then they picked up one of their pens to scribble with. So, using the explanatory proviso to the POI, there is an equal probability that Annie and Bob each scribbled in the book. Annie has more pens than Bob, so it's less likely that any one of her pens was used to scribble in the book. Similarly, in the *Buses Murder* case, which CEO decided to do the murder plot is explanatorily prior to which bus they crashed. They first plotted the murder, and then they chose one of the buses to crash. So, using the explanatory proviso to the POI, there is an equal probability that the Blue Bus and Red Bus company crashed a bus.

In civil lawsuits, which company is responsible for the crash will often be explanatorily prior to which bus crashed. The company may be liable because of some intentional or negligent act that led to the crash, which is in the case of the examples we have already seen, including *Murder*. The company may also be liable under the *respondeat superior* rule, which could allow the pedestrian to sue the company because of the actions of the driver (Silver, 2023). On this legal doctrine, an employer can be held responsible for the actions of an employee, when they have been acting in the scope of their role under the direction of the employee, much the same way that parents may be held responsible for property damage done by their children. The idea in the parenting case is that the parent has (or ought to have) influence over their child such that bad behavior on the child's part reflects a mistake on the part of the parents. If a child throws a rock through a neighbor's window, the parents are responsible for the damage because they shouldn't have let the child throw rocks. In the employment case, the company could have done more to prevent the crash: hire better drivers, provide better training, and so on. So, tacit in the very use of the *respondeat superior* rule is the fact that the actions the bus company are explanatorily prior to which bus crashed.

II. The Principle of Indifference, Priors, and Posteriors

What if the cases so far are too simple? Maybe each time a bus goes out on the road, there is some chance that a bus crashes, and so the more buses on the road, the more likely it is that there's a crash. Then, because the Blue Bus company has more buses than the Red Bus company, they have more opportunities to crash. If someone dies in a game of Russian roulette, it's more likely that the bullet was from the gun loaded with 5 bullets than the gun loaded with 1 bullet. If we think about the bus fleet as a series of coin flips, a roll of the dice, or a game of Russian roulette, then surely, it's more likely that the crash came from the larger bus company, which had so many more opportunities to cause a crash. Here's *Buses* again, with one extra bit of information.

> **Buses #2** A bus crashes into a pedestrian. *The crash was caused by a company policy (held by both of the companies) that results in a 5% chance that each bus crashes*. 100 buses operate on the route. 80 buses are operated by the Blue Bus Company and 20 buses are operated by the Red Bus Company.

Let B denote that that *the bus is blue* and C denote that *the bus crashed*. Then,

$P(B|C) = P(B \text{ and } C)/P(C) = P(B)*P(C)/P(C) = .8.$

In this case, we can stick with the original idea that there is an 80% probability that the Blue Bus company is responsible for the crash. So far, so good for the standard treatment of the case. This is, however, just one way we can spell out the details. Here's another way to spell out the details, and it's one that has counterintuitive results. It ends up being the case that the Red Bus company caused the crash.

> **Buses #3:** A bus crashes into a pedestrian. *This was the only bus on the route that crashed*. The crash was caused by a company policy that results in a 5% chance of each bus that operates on the route to crash. *Exactly one bus has the policy*. 100 buses operate on the route. 80 buses are operated by the Blue Bus company and 20 buses are operated by the Red Bus company.

Let B denote that the *Blue Bus company has the policy*, R denote that the *Red Bus company has the* policy, and E denote that *exactly one bus crashed*. The most explanatorily fundamental level is which company has the policy, so in the absence of other information we should start with $P(B) = P(R)$. Then, say that we learn that exactly one bus crashed into a pedestrian. The probability that the Blue Bus company holds the policy and caused the crash can be given as follows:

$P(B \mid E) = P(E \mid B) / [P(E \mid B) + P(E \mid R) = (.070)/(.070 + .377) = .16$

So in this case, it's actually improbable that the Blue Bus company is responsible for the crash (the intuitive explanation: if the Blue Bus company had this policy, we would expect more than one crash!)

> **Buses #4:** A bus crashes into a pedestrian. *This was the only bus on the route that crashed.* The crash was caused by a company policy that resulted in a 1% chance of each bus that operates on the route to crash. *Exactly one bus has the policy*. 100 buses operate on the route. 80 buses are operated by the Blue Bus Company and 20 buses are operated by the Red Bus Company.

Again, the most explanatorily fundamental fact is which of the Red Bus company or the Blue Bus company has the policy. So, start with the same prior probability that the Red Bus and Blue Bus company have the policy. The probability that the Blue Bus company holds the policy and caused the crash can be given as follows:

$P(B \mid E) = P(B \mid E) = P(E \mid B) / [P(E \mid B) + P(E \mid R) = (.36)/(0.36+ 0.17) = .68$

So in this case, again, the evidence suggests that the Blue Bus company is responsible, although not to the same degree as the .8 probability that it's typically claimed to support.

The original *Buses* case is woefully underspecified. So far, we have seen four ways to spell out the details: *Murder, and Buses #1, #2, and #3*, but this is in no way comprehensive. We could keep coming up with cases *ad nauseum*, with information and uncertainty about the number of buses in each fleet; the chances each bus crashes given a policy; whether at least one or exactly one or both companies have the policy; the total number of buses that crashed, and so on. All of the parameters will matter to the probability of that a Blue Bus or Red Bus crashed. It's a headache. In very similar ways of spelling out the details of the case, with very subtle differences between the cases, the probability that the bus belongs to the Red Bus and Blue Bus van vary drastically. In some of these cases, the Blue Bus company is probably not responsible, and it takes some mathematics to tell the difference between these cases and the cases in which the Blue Bus company is responsible. It follows that we should not rest our legal epistemology on the *Buses* case. It's not clear at all whether the reluctance to convict the Blue Bus company on this evidence is due to the statistical nature of the evidence, or the fact that it's practically indistinguishable from cases in which the evidence supports a high probability that the Red Bus company is responsible.

III. Toxic Torts and the Principle of Indifference

Legal epistemologists often note that there are discrepancies in how courts use merely statistical evidence (Moss, 2020). In some cases, it seems as though merely statistical evidence can appropriately be used to find an entity liable, based on the actual *Kramer v. Weedhopper*.

> **Bolts:** A bolt manufacturer sells a defective bolt to an airplane company. The bolt snaps and an airplane crashes. Manufacturer A supplied 90% of the bolts and Manufacturer B supplied 10% of the bolts.

Unlike in *Buses*, this evidence *Bolts* is enough to find Manufacturer A liable for the defective bolt.

The case against the bolt manufacturer one of *strict liability*, which means that the manufacturer can be held responsible without proof of negligence, intent, or knowledge on the part of the manufacturer.[2] The doctrine of strict liability weaves together a few legal principles. Sometimes, strict liability is justified because it's just too hard to prove that there was intent, knowledge, or negligence on the part of the manufacturer, but the victims still need compensation, and so a conviction can be reached on this less-than-ideal evidence. Sometimes, strict liability is justified because some mistakes must be due to negligence. If you have caused the harm, it must be your fault (an example: statutory rape.) Sometimes, strict liability is seen as recognizing a risk of causing harm, for which one may be liable, as a "cost of doing business." If a company manufactures bolts, there's some chance that one of their bolts has an air bubble, even if they have the best manufacturing practices. When a plane comes down, the victims need compensation, and

[2] See Epstein (1973) for a classic defense of the strict liability doctrine.

so it comes from the company. On this final picture, there is no act from manufacturer A or B that is explanatorily prior to the bolt's defect. So, this looks like *Buses #2*, and there is a .9 probability that manufacturer A produced the defective bolt.

So, there is an important difference between the use of this evidence in the strict liability cases and in the non-strict liability cases, and that's in the strict liability cases, the statistical evidence actually supports a high probability that the defendant is guilty.

IV. Conclusion

What's the upshot? The *Buses* case, which has been used to illustrate the problem of merely statistical evidence a thousand times, just doesn't work. The problem of merely statistical evidence is still a problem, which we can see from cases like this (Nesson, 1976):

> *Prisoners*: 24/25 prisoners incarcerated on a cell block riot and attack a guard. One innocent prisoner runs and hide. The riot is caught on tape, but the prisoners are wearing identical jumpsuits and it's too grainy to identify any of them by their faces. The prosecutors select a prisoner at random and charge him with murder.

The evidence in this case really does support a .96 probability that the defendant is guilty. The evidence does not meet the burden of proof in criminal law, and it probably doesn't meet the burden of proof in civil law either.

Nevertheless, once we no longer need a theory of legal proof that explains why the evidence in *Buses* does not count as legal proof, conceptual space open up. Here's just one example. Suppose a juror can only convict a defendant if they *think* that that defendant in particular did it, that is, if it's more likely that the defendant stand did it than any other suspect.[3] In the *Prisoners* case, you do not think that the suspect did it. In the *Buses* case, if we stuck with the original understanding of the case, one should think that the Blue Bus Company is responsible.

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[3] This is the notion of *think* introduced by Holguin (2021).

Citations [Under Construction]

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